May 22.

Rev. B. LLOYD, D. D., Provost, T. C. D., President, in the Chair.

Rev. James Horner, D. D., was elected member of the Academy.

Professor Mac Cullagh read a letter from Joseph S. Moore, Esq. on the Australian instrument called *kilee* or *boomerang*, so remarkable for the course that it takes when thrown in the air. It is a flat piece of wood of a hyperbolic form, about 2½ inches broad, perfectly plane on one side, and slightly convex on the other. A right line joining its extremities is about two feet long, and the middle of this right line is distant about a foot from the middle of the instrument or the vertex of the hyperbola. When properly thrown, it makes a circuit, returns, passes close to the person who threw it, and even goes behind him, and then attempts to return again before it falls to the ground. It is curious that such a missile should have been invented by savages, for, as far as we know, it is found only among the natives of New Holland. It is said to be called *kilee* on the western, and *boomerang* on the eastern coast of that country. Some of these *kilees* had been sent to Mr. Moore from the Swan River, and though he was unsuccessful in throwing them, he succeeded with others which he caused to be made of the same general form, but much more curved than the originals. The dimensions given above are those which he found most convenient. The following is an extract from Mr. Moore's letter:

"The natives throw them with the convex edge against the air; their movement is then from left to right. But the way in which I have succeeded was by taking the missile by

one end, with the concave edge inward, and the plane side undermost, the plane making an angle of about forty degrees with the horizon; throwing as if to strike the ground at the distance of about thirty yards, and giving it, on leaving the hand, a rapid rotatory as well as progressive motion. Instead of striking the ground at which it was aimed, its plane becomes horizontal at the distance of twenty-five yards, and so continues for about fifteen yards, when it commences rising in the air, and moving towards the left; its plane then becomes inclined, and continues at an angle of from thirty to forty degrees, whilst it describes apparently a segment of a circle to the left. Having, at the distance of sixty or seventy yards, attained an altitude of from forty to sixty feet, the projectile returns, descending to the point from which it was projected, when its plane becoming once more horizontal, it skims along within a few feet of the ground, and passes close by the right hand of the person who threw it. On passing, its plane becomes elevated once more, it rises a second time, and performs another smaller curve, (fifteen or twenty yards behind the projector,) in like manner as the first, with this singular exception, that the second curve is described from left to right, contrary to the course of its rotation and of the first curve, which is invariably from right to left."

In bringing the instrument under the notice of the Academy, Mr. Mac Cullagh wished to draw attention to the theory of its motion. When a body of any form whatsoever is projected in vacuo, we know that its centre of gravity must describe a parabola in a vertical plane, while the body spins about an axis passing through that centre. In the present case, therefore, it is clear that the continued swerving from the vertical plane must be ascribed to the action of the air. But to compute accurately the mutual action of the air, and of a body endued, at the same time, with a progressive and a rotatory motion, is a problem far beyond the present powers of science. The problem can only be solved approximately;

and, however we may simplify it, the calculations are likely to be very troublesome. Even the supposition of a resistance proportional to the square of the velocity (which is usually considered as an approximation in questions of this sort), would lead to complicated results. It may be observed, that the motion of the *kilee* is rudely imitated in the familiar experiment with a card, cut into the form of a crescent, and sent off by a fillip, so as to spin in its own plane.

Mr. Petrie read the concluding portion of his paper "On the Antiquities of Tara Hill."

In this, as well as in the preceding part, the author has endeavoured to ascertain from historic evidences, not only the period to which each of the monuments now remaining should be referred, but also the date of those of which no vestiges exist, but whose features and localities are described in ancient documents. In this investigation, the author brought forward a great number of ancient Irish authorities not hitherto used or translated, of which one of the most curious and interesting is a description of the banqueting hall or house of assembly, written by Cinaeth O'Hartigan, a celebrated poet of the tenth century. From all these documents it appears, that, with the exception of the original Tuatha Dedanann cahir, and coronation stone, all the monuments now or formerly existing on Tara Hill may be classed under two distinct eras, both within the limit of authentic Irish history. The first and less important class comprises the monuments belonging to the age of the hero Cuchullin, who died in the early part of the first century; and of these there are no remains. The second—to which nearly all the existing monuments belong—extends to the time of the monarch Cormac Mac Art, in the third century. There are only two or three monuments of later date. From these facts, the author concludes, that, before the latter period, Tara had attained to

no distinguished celebrity as a regal city; and hence its omission from the map of Ptolemy, who wrote in the century preceding.

Another fact, derived both from historic evidences and existing remains, is, that, with the exception of the cahir erected by the Tuatha Dedananns, all the works appear to have been of earth and wood; though forts and houses of uncemented stones are found in other districts of equally ancient or even earlier date. From the uniform character which pervades these remains, the author concludes that they are the monuments of *one* people; and he thinks that the fact above mentioned may help to elucidate the origin of that Scotic race, which ruled in Ireland at the period of their construction.

Sir William Hamilton laid before the Academy an account of some investigations, in which he had recently been engaged, respecting Equations of the Fifth Degree. They related chiefly to three points: first, the argument of Abel against the possibility of generally and algebraically resolving such equations; second, the researches of Mr. Jerrard; and third, the conceivable reduction, in a new way, of the original problem to a more simple form.

1. The argument of Abel consisted of two principal parts; one independent of the degree of the equation, and the other dependent on that degree. The general principle was first laid down, by him, that whatever may be the degree n of any general algebraic equation, if it be possible to express a root of that equation, in terms of the coefficients, by any finite combination of rational functions, and of radicals with prime exponents, then every radical in such an expression, when reduced to its most simple form, must be equal to a rational (though not a symmetric) function of the n roots of the original equation; and must, when considered as such a function, have exactly as many values, arising from the

permutation of those n roots among themselves, as it has values, when considered as a radical, arising from the introduction of factors which are roots of unity. And in proceeding to apply this general principle to equations of the fifth degree, the same illustrious mathematician employed certain properties of functions of five variables, which may be condensed into the two following theorems: that, if a rational function of five independent variables have a prime power symmetric, without being symmetric itself, it must be the square root of the product of the ten squares of differences of the five variables, or at least that square root multiplied by some symmetric function; and that, if a rational function of the same variables have, itself, more than two values, its square, its cube, and its fifth power have, each, more than two values also. Sir W. H. conceived that the reflections into which he had been led, were adapted to remove some obscurities and doubts which might remain upon the mind of a reader of Abel's argument; he hoped also that he had thrown light upon this argument in a new way, by employing its premisses to deduce, *à priori*, the known solutions of quadratic, cubic, and biquadratic equations, and to show that no new solutions of such equations, with radicals essentially different from those at present used, remain to be discovered: but whether or no he had himself been useful in this way, he considered Abel's result as established: namely, that it is impossible to express a root of the general equation of the fifth degree, in terms of the coefficients of that equation, by any finite combination of radicals and rational functions.

2. What appeared to him the fallacy in Mr. Jerrard's very ingenious attempt to accomplish this impossible object, had been already laid before the British Association at Bristol, and was to appear in the forthcoming volume of the reports of that Association. Meanwhile, Sir William Hamilton was anxious to state to the Academy his full conviction, founded

both on theoretical reasoning and on actual experiment, that Mr. Jerrard's method was adequate to achieve an almost equally curious and unexpected transformation, namely, the reduction of the general equation of the fifth degree, with five coefficients, real or imaginary, to a trinomial form ; and therefore ultimately to that very simple state, in which the sum of an unknown number, (real or imaginary), and of its own fifth power, is equalled to a known (real or imaginary) number. In this manner, the general dependence of the modulus and amplitude of a root of the *general* equation of the fifth degree, on the five moduli and five amplitudes of the five coefficients of that equation, is reduced to the dependence of the modulus and amplitude of a new (real or imaginary) number on the one modulus and one amplitude of the sum of that number and its own fifth power; a reduction which Sir William Hamilton regards as very remarkable in theory, and as not unimportant in practice, since it reduces the solution of any proposed numerical equation of the fifth degree, even with imaginary coefficients, to the employment, without tentation, of the known logarithmic tables, and of two new tables of double entry, which he has had the curiosity to construct and to apply.

3. It appears possible enough, that this transformation, deduced from Mr. Jerrard's principles, conducts to the simplest of all forms under which the general equation of the fifth degree can be put; yet, Sir William Hamilton thinks, that algebraists ought not absolutely to despair of discovering some new transformation, which shall conduct to a method of solution more analogous to the known ways of resolving equations of lower degrees, though not, like them, dependent entirely upon radicals. He inquires in what sense it is true, that the general equation of the fifth degree would be resolved, if, contrary to the theory of Abel, it were possible to discover, as Mr. Jerrard and others have sought to do, a reduction of that general equation to the

binomial form, or to the extraction of a fifth root of an expression in general imaginary? And he conceives, that the propriety of considering such extraction as an admitted instrument of calculation in elementary algebra, is ultimately founded on this: that the two real equations,

$$x^5 - 10\,x^3 y^2 + 5\,xy^4 = a,$$
$$5\,x^4 y - 10x^2 y^3 + y^5 = b,$$

into which the imaginary equation

$$(x + \sqrt{-1}\,y)^5 = a + \sqrt{-1}\,b$$

resolves itself, may be transformed into two others which are of the forms

$$\rho^5 = r, \text{ and } \qquad \frac{5\tau - 10\tau^3 + \tau^5}{1 - 10\tau^2 + 5\tau^4} = t,$$

so that each of these two new equations expresses one given real number as a known rational function of one sought real number. But, notwithstanding the interest which attaches to these two particular forms of rational functions, and generally to the analogous forms which present themselves in separating the real and imaginary parts of a radical of the n^{th} degree; Sir William Hamilton does not conceive that they both possess so eminent a prerogative of simplicity as to entitle the inverses of them alone to be admitted among the instruments of elementary algebra, to the exclusion of the inverses of all other real and rational functions of single real variables. And he thinks, that since Mr. Jerrard has succeeded in reducing the general equation of the fifth degree, with five imaginary coefficients, to the trinomial form above described, which resolves itself into the two real equations following,

$$x^5 - 10x^3 y^2 + 5xy^4 + x = a,$$
$$5x^4 y - 10x^2 y^3 + y^5 + y = b,$$

it ought now to be the object of those who interest themselves in the improvement of this part of algebra, to inquire,

whether the dependence of the two real numbers x and y, in these two last equations, on the two real numbers a and b, cannot be expressed by the help of the real inverses of some new real and rational or even transcendental functions of single real variables; or, (to express the same thing in a practical, or in a geometrical form,) to inquire whether the two sought real numbers cannot be calculated by a finite number of tables of single entry, or constructed by the help of a finite number of curves: although the argument of Abel excludes all hope that this can be accomplished, if we confine ourselves to those particular forms of rational functions which are connected with the extraction of radicals.

DONATIONS.

Voices from the Time of the Reformation of the Danish Church. (In Danish.) Presented by the Literary Society of the Diocese of Funen.

Observations on some of the Strata between the Chalk and Oxford Oolite, in the South-East of England. By William Henry Fitton, M.D., F.R.S., &c. Presented by the Author.